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Duality Theorem A Tool For Quick Solution in Decision Making

Adedoyin .I.S, Adeoye A. O., Olatinwo .M.
Department of Business Administration Kwara State Polytechnic, Ilorin, Nigeria
Department of Statistics Federal Polytechnic offa Kwara State Nigeria
Department of Mathematics Federal Polytechnic offa Kwara State Nigeria

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ABSTRACT

Linear programming problem is a mathematical technique for finding the best used of an organization resource , duality is a method of finding the solution to a linear programming problem. When considered linear programming problem with two variables using simplex method, the optimal solution is obtained at third iterations. When the problem is changed to dual problem and simplex method was used to obtained the optimal solution, the optimal solution is obtained at first iteration, and all information needed in the original problem are obtained in the dual result . This implied that dual problem has reduced the burden of computation by one iteration and all information needed in the original problem will be obtain at the optimal solution. This approach saves time and minimizes the cost incurred for estimating the maximum number of product expected to be produced by the management of industry.

Keywords: Duality theorem a better approach in linear programming

I. INTRODUCTION

Duality In The Linear Programming Problem

Every Linear Programming Problem is associated with another Linear Programming Problem call the **DUAL** of the problem. The original problem is called **PRIMAL**, while the other is called **DUAL**. The optimal solution of either problem reveals' information concerning the optimal solution of the other. If the optimal solution of either primal or dual is known 'the optimal solution of the other is also available.

Rules of changing primal problem to Dual problem

i. Changing the variables from X_1 to any convenient variable say W.
ii. Transposing the coefficients matrix A
iii. Interchanging the role of constant terms coefficients of the objectives function
iv. Reverting the inequality and
v. Minimizing the objective function instead of maximizing it.

Methodology

Given a linear programming problem (LPP)

$$\text{Optimize } z = \underline{C}^T \underline{x}$$
$$s.t. \quad A\underline{x} * \underline{b}$$
$$\underline{x} \geq 0$$

To expand the above equation we have

$$ie, Optimize Z_p = c_1x_1 + c_2x_2 + \ldots + c_nx_n \quad \ldots\ldots\ldots(1)$$

$$s.t.$$

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \quad * b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n \quad * b_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n \quad * b_m \end{array}\right\}$$

Where * assume, =, ≤, ≥.

The original general problem is called Primal problem.

When the primal problem is change to dual problem, we have the following

Optimize

$$z_D = \underline{b}^T w$$
$$s.t. \quad A^T w \quad * \quad \underline{c}$$
$$\underline{w} \geq 0$$

$*$ assume, $=$, $\leq$, $\geq$.
The above can be expanded as follows.
Optimize

$$z_D = b_1w_1 + b_2w_2 + \ldots + b_nw_n$$

$$s.\,t.$$

$$\begin{array}{ll} a_{11}w_1 + a_{12}w_2 + a_{13}w_3 + \ldots + a_{1n}w_n & * \; c_1 \\ a_{21}w_1 + a_{22}w_2 + a_{23}w_3 + \ldots + a_{2n}w_n & * \; c_2 \\ \vdots & \vdots \\ a_{m1}w_1 + a_{m2}w_2 + a_{m3}w_3 + \ldots + a_{mn}w_n & * \; c_m \\ w_j \geq 0 & \end{array}$$

Where $*$ assume, $=$, $\leq$, $\geq$.
For a case of two variable we have optimize

$$z_p = c_1x_1 + c_2x_2$$

$$s.\,t.$$

$$\begin{array}{ll} a_{11}x_1 + a_{12}x_2 & * \; b_1 \\ a_{21}x_1 + a_{22}x_2 & * \; b_2 \\ \vdots & \vdots \\ a_{m1}x_1 + a_{m2}x_2 & * \; b_m \\ \underline{x} \geq 0 & \end{array}$$

Changing the above to dual problem we have optimize

$$z_p = b_1w_1 + b_2w_2 + \ldots + b_nw_n$$

$$s.\,t.$$

$$\begin{array}{ll} a_{11}w_1 + a_{21}w_2 + \ldots + a_{m1}w_m & * \; c_1 \\ a_{12}w_1 + a_{22}w_2 + \ldots + a_{m2}w_m & * \; c_2 \\ \underline{w} \geq 0. & \end{array}$$

We will observe that we normally apply elementary row operation for the solution of linear programming problem. z_P has m rows, while z_D has two rows, which make our computations to be easy.

Application

Given a linear programming problem with two variables

$$\begin{array}{l} Max \;\; z_p = 3x_1 - 2x_2 \\ s.\,t. \quad x_1 \leq 4 \\ x_1 + x_2 \leq 6 \\ x_1 + x_2 \leq 5 \\ x_2 \leq 1 \\ x_1, \; x_2 \geq 0. \end{array}$$

In other to solve the primal problem above, the slack variables need to be added. Adding slack variable for the primal problem becomes;

$$\begin{array}{l} Max \;\; z_p = 3x_1 - 2x_2 \\ x_1 \quad + x_3 = 4 \\ x_1 + x_2 + x_4 = 6 \\ x_1 + x_2 + x_5 = 5 \\ -x_2 - x_6 = 1 \\ \underline{x} \geq 0. \end{array}$$

The first table of the above problem is ;

BV	CB	x_1	x_2	x_3	x_4	x_5	x_6	xB_i
x_3	0	1	0	1	0	0	0	4
x_4	0	1	1	0	1	0	0	6
x_5	0	1	1	0	0	1	0	5
x_6	0	0	-1	0	0	0	1	1
$z_j - c_j$		-3	2	0	0	0	0	$z = 0$

To go from the initial table to the next, we perform row operations, but in the 2nd table, most zeros will vanish and most of the elements of the 2nd table will be fraction. We observe again that the basic is about 4 by 4 matrix before we can get the optimal solution we would have many iterations. For the dual problem of the primal problem, linear programming problem becomes.

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$$\min z_D = 4w_1 + 6w_2 + 5w_3 + w_4$$
$$s.t. \quad w_1 + w_2 + w_3 + 0w_4 \geq 3 \quad ....(i)$$
$$0w_1 + w_2 + w_3 + w_4 \geq -2 \quad ....(ii)$$
$$w \geq 0.$$

Multiply the objective function by -1 and introducing the surplus variable, then the equation becomes

$$Max \quad z'_D = -4w_1 - 6w_2 - 5w_3 - w_4$$
$$s.t. \quad w_1 + w_2 + w_3 + 0w_4 - w_5 = 3$$
$$0w_1 - w_2 - w_3 + w_4 - w_6 = 1$$
$$\underline{w} \geq 0.$$

Then the first simplex table becomes ;

BV	CB	w_1	w_2	w_3	w_4	w_5	w_6	w_B
w_1	- 4	1	1	1	0	-1	0	3
w_4	-1	0	-1	-1	1	0	-1	2
$z_j - c_j$		0	3	2	0	4	1	.

Since $z_j - c_j \geq 0$, the solution is optimal at first table with

$w_1 = 3, \quad w_4 = 2$,

$$z'_D = -12 + 2 = -10$$
$$z'_D = 10$$

To obtain variable $\underline{x}$ from the dual table we look for the slack or surplus variable at the $z_j - c_j$ row of the optimal table above. The first slack or surplus variable correspond to the first variable x_1, while second one correspond to the variable x_2. Substituting from the objective function z_p of primal problem

$x_1 = 4, \quad x_2 = 1$

$$z_p = 3x_1 - 2x_2$$
$$z_p = 3(4) - 2(1) = 10.$$

II. CONCLUSION

The solution obtained using dual problem has reduced the burden of computation from one iteration to the next. This approach saves time and minimize the cost of incurred for estimating the maximum number of products expected to be produced by any company .

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